Chapter 2

Preliminaries

§2.1.   Two lemmas on Riccati's differential equations

First of all, we give two lemmas on ordinary differential equations of Riccati's type. These two lemmas are due to L. Hörmander [Ho1].

Lemma 2.1.   Let $z = z(t)$ be a solution in [0,T] of the Riccati's differential equation:

$$\frac{dz}{dt} = a_0(t)z^2 + a_1(t)z + a_2(t), \qquad (2.1.1)$$

where $a_j(t)$ $(j = 0, 1, 2)$ are continuous, $a_0(t) \geq 0$, and $T > 0$ is a given real number. Let

$$K = \int_0^T |a_2(t)| \, dt \cdot \exp\left(\int_0^T |a_1(t)| \, dt\right). \qquad (2.1.2)$$

If

$$z(0) > K, \qquad (2.1.3)$$

then it follows that

$$\int_0^T a_0(t)dt \cdot \exp\left(-\int_0^T |a_1(t)| \, dt\right) < (z(0) - K)^{-1}. \qquad (2.1.4)$$

$\square$