Chapter 1

Introduction

We are interested in the following quasilinear hyperbolic system of balance laws

$$\frac{\partial u}{\partial t} + A(u)\frac{\partial u}{\partial x} = B(u), \tag{1.1}$$

where $u = (u_1, \cdots, u_n)^T$ is the unknown vector function standing for the density of physical quantities, $A(u) = (a_{ij}(u))$ is an $n \times n$ matrix with suitably smooth elements $a_{ij}(u)$ $(i, j = 1, \cdots, n)$, it represents the gradient matrix of the flux function, $B(u) = (B_1(u), \cdots, B_n(u))^T$ is a given smooth vector function denoting for the source term. System (1.1) describes many physical phenomena. In particular, important examples occur in gas dynamics, shallow water theory, plasma physics, combustion theory, nonlinear elasticity, acoustics, classical or relativistic fluid dynamics and petroleum reservoir engineering (see [An], [CF], [CM], [LL], [Se], [Ta], etc.). These equations play an important role in both science (such as physics, mechanics, biology, etc.) and technology.

By hyperbolicity, for any given u on the domain under consideration, $A(u)$ has n real eigenvalues $\lambda_1(u), \cdots, \lambda_n(u)$ and a complete system of left (resp. right) eigenvectors. For $i = 1, \cdots, n$, let $l_i(u) = (l_{i1}(u), \cdots, l_{in}(u))$ (resp. $r_i(u) = (r_{i1}(u), \cdots, r_{in}(u))^T$) be a left (resp. right) eigenvector corresponding to $\lambda_i(u)$:

$$l_i(u)A(u) = \lambda_i(u)l_i(u) \quad (\text{resp. } A(u)r_i(u) = \lambda_i(u)r_i(u)). \tag{1.2}$$