Chapter 7

Proof of the Orbifold Theorem

In this section we will give a sketch of the proof of the Orbifold Theorem. We have assumed that the reader is familiar with the definitions and general ideas presented so far in this memoir and have tried to highlight the main theorems needed in the proof of this result. For a complete proof the reader may consult [19]. For an alternative proof of a somewhat different version of the Theorem, see [8].

7.1 Topological preliminaries

The Orbifold Theorem states that if a compact, orientable, orbifold-irreducible orbifold has a 1-dimensional singular locus, then it can be cut along a (possibly empty) Euclidean 2-orbifold so that each of the resulting components has a geometric structure. As discussed in Chapter 2, this is the orbifold version of the Geometrization Conjecture 2.57 for orientable 3-orbifolds with the important assumption that the singular locus is non-empty.

Theorem 7.1 (The Orbifold Theorem).
Suppose that $\mathcal{O}$ is a compact, orientable, orbifold-irreducible 3-orbifold with (possibly empty) orbifold-incompressible boundary consisting of Euclidean 2-orbifolds. Suppose that $\Sigma(\mathcal{O})$ is a non-empty graph. Then there is an incompressible Euclidean 2-suborbifold $\mathcal{T}$ (possibly empty) such that each component of $\mathcal{O} - \mathcal{T}$ is a geometric orbifold.

To begin the proof of the Orbifold Theorem, we need an orbifold version of the torus decomposition of a 3-manifold. This will provide the incom-