Chapter 1

Geometric Structures

The aim of this memoir is to give an introduction to the statement and main ideas in the proof of the "Orbifold Theorem" announced by Thurston in late 1981 ([83], [81]). The Orbifold Theorem shows the existence of geometric structures on many 3-dimensional orbifolds, and on 3-manifolds with a kind of topological symmetry.

The main result implies a special case of the following Geometrization Conjecture proposed by Thurston in 1976 as a framework for the classification of 3-manifolds. For simplicity, we state the conjecture only for compact, orientable 3-manifolds.

Conjecture 1.1 (Geometrization Conjecture). ([81]) *The interior of every compact 3-manifold has a canonical decomposition into pieces having a geometric structure.*

The kinds of *decomposition* needed are:

1. prime (or connected sum) decomposition, which involves cutting along separating 2-spheres and capping off the pieces by gluing on balls.
2. torus decomposition, which involves cutting along certain incompressible non-boundary parallel tori.

The meaning of *canonical* is that the pieces obtained are unique up to ordering and homeomorphism. The spheres used in the decomposition are not unique up to isotopy, but the tori are unique up to isotopy.

A *geometric structure* on a manifold is a complete Riemannian metric which is locally homogeneous (i.e. any two points have isometric neighbourhoods). A *geometric decomposition* is a decomposition of this type into