Hence, to obtain the required formula, it is sufficient to show

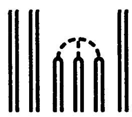
$$\iota_d \check{Z}(U_+)^{3d} = (-1)^d + (\text{terms of degree} > 0), \tag{7.9}$$

$$\iota_d \check{Z}(\mathcal{L}_D) = D + (\text{terms of degree} > d). \tag{7.10}$$

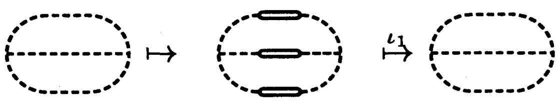
For the proof of (7.9), see [27]. Further we obtain (7.10) by Lemma 7.12 below.

□

Lemma 7.12.

$\check{Z}(\mathcal{L}_D) =$  $+$ (terms of $\sharp$ { trivalent vertices } ≥ 2)

Proof. We obtain the formula by long calculation along the definition of $\hat{Z}$. For example, for the dashed θ curve D, we show rough pictures of the calculation below. Recall that $\mathcal{L}_D$ is a linear sum of links with 3 components in this case (with $3d$ components in general).



D $\quad$ $\hat{Z}(\mathcal{L}_D) \sim \check{Z}(\mathcal{L}_D)$

For the detailed proof, see [22]. □

8 Quantum invariants and the universal perturbative invariant

8.1 Quantum $SO(3)$ invariant constructed from quantum invariants of framed links

Let V_m be the m dimensional irreducible representation of sl_2 and M the 3-manifold obtained from S^3 by Dehn surgery along a framed link L.

Theorem 8.1 ([12]). Let r be an odd integer ≥ 3, and put $q = \exp(2\pi\sqrt{-1}/r)$. Then

$$\frac{\sum [m] Q^{sl_2;V_m}(L)}{\left(\sum [m] Q^{sl_2;V_m}(U_+)\right)^{\sigma_+} \left(\sum [m] Q^{sl_2;V_m}(U_-)\right)^{\sigma_-}} \in \mathbb{C}$$