



Figure 5.1: Three kinds of invariants of knots and the relations between them with the weight system $\hat{W}_{g,R}$ derived from the substitution of g and R into chord diagrams. The other is the universality among Vassiliev invariants; each Vassiliev invariant v is expressed as

$$v = W \circ \hat{Z}$$

with some weight system W .

As a corollary of the two universalities, we obtain a relation between quantum invariants and Vassiliev invariants; the coefficients of the quantum (g, R) invariant are Vassiliev invariants and their weight systems are equal to $W_{g,R}$.

6 The universal perturbative invariant of 3-manifolds

So far we have dealt with invariants of knots and links. From now on we will consider invariants of 3-manifolds. The purpose of this section is to construct an invariant of 3-manifolds which has the universal property that the perturbative quantum invariants of 3-manifolds recover from it. So we call it the *universal perturbative invariant* of 3-manifolds.

6.1 Properties of $\hat{Z}(L)$

We will construct invariants from $\hat{Z}(L)$ in Section 6.4. To show the invariance under Kirby moves, we need the following properties of $\hat{Z}(L)$.