$v : \mathcal{K}_d \to \mathbb{C}$. Further this map induces a linear map $\mathcal{K}_d/\mathcal{K}_{d+1} \to \mathbb{C}$, since v is of degree d *i.e.*, $v|_{\mathcal{K}_{d+1}} = 0$. By composing φ, we obtain the weight system W_k of v. Further we have the inverse $[\hat{Z}]$ of φ by Theorem 4.5. These maps are written in a diagram as:

$$\begin{array}{cccccc} \mathcal{K} & \supset & \mathcal{K}_d & \xrightarrow{v} & \mathbb{C} \\ & & \text{projection} \downarrow & \circlearrowleft \nearrow \circlearrowleft & \uparrow W_d \\ & & \mathcal{K}_d/\mathcal{K}_{d+1} & \overset{\varphi}{\underset{[\hat{Z}]}{\overleftarrow{\underset{\circlearrowleft}{\longrightarrow}}}} & \mathcal{A}(S^1)^{(d)} \end{array}$$

where we obtain the commutativity of this diagram by Theorem 4.5 and the definition of the weight system. Thus we have $(v - W_d \circ [\hat{Z}])|_{\mathcal{K}_d} = 0$ in the diagram. Hence the map

$$v - W_d \circ \hat{[Z]} : \{\text{knots}\} \to \mathbb{C}$$

is a Vassiliev invariant of degree $d-1$. We put W_{d-1} to be the weight system of $v - W_d \circ [\hat{Z}]$.

For $k = d-2$, we put W_{d-2} to be the weight system of $v - (W_d + W_{d-1}) \circ [\hat{Z}]$; it is a Vassiliev invariant of degree $d-2$ by the same argument as above.

For $k = d-3, d-4, \cdots$, we can go on similarly for the rest. □

5 Vassiliev invariants and quantum invariants

We have seen the relations between quantum invariants and the modified Kontsevich invariant in Section 3, and between Vassiliev invariants and the modified Kontsevich invariant in Section 4. In this section, we see a relation between quantum invariants and Vassiliev invariants.

Theorem 5.1 **([4]).** For a framed knot K, the coefficient of h^d in $Q^{\mathfrak{g},R}(K)|_{q=e^h}$ is a Vassiliev invariant of degree d as an invariant of K.

Proof. In a construction of $Q^{\mathfrak{g},R}(K)|_{q=e^h}$, we associate positive and negative crossings with R-matrices $\mathcal{R}_+$ and $\mathcal{R}_-$, respectively. These two R-matrices