completing this case.

Lastly, we show that the invariant $\hat{W}(\hat{Z}(L))$ satisfies the third formula (3.10), using formulas obtained above. We have

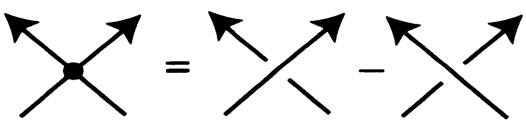
$$
\begin{aligned}
&(q^{1/2} - q^{-1/2})\hat{W}(\hat{Z}(\uparrow))\hat{W}(\hat{Z}(\bigcirc)) \\
&= (q^{1/2} - q^{-1/2})\hat{W}(\hat{Z}(\uparrow \bigcirc)) \\
&= q^{1/2N}\hat{W}(\hat{Z}(\uparrow\!\!\bigcirc)) - q^{-1/2N}\hat{W}(\hat{Z}(\uparrow\!\!\bigcirc)) \\
&= q^{1/2N}q^{(N-1/N)/2}\hat{Z}(\uparrow) - q^{-1/2N}q^{-(N-1/N)/2}\hat{Z}(\uparrow) \\
&= (q^{N/2} - q^{-N/2})\hat{Z}(\uparrow).
\end{aligned}
$$

Hence we have $\hat{W}(\hat{Z}(\bigcirc)) = [N]$. □

4 The modified Kontsevich invariant and Vassiliev invariants

4.1 Vassiliev invariants of framed knots

We denote framed knots with even[5] framings simply by knots in this section. Let $\mathcal{K}$ be the vector space freely spanned by knots over $\mathbb{C}$. A *singular knot* is an immersion of S^1 into S^3 whose singularities are transversal double points. We regard a singular knot as an element in $\mathcal{K}$ by linearly removing each singularity by the following relation



for example, see Figure 4.1. We define the subspace $\mathcal{K}_d$ of $\mathcal{K}$ by

$$\mathcal{K}_d = span\{\text{the singular knots with } d \text{ singular points}\}.$$

[5]The framing of a framed knot usually changes by even, by a crossing change. Hence we consider framed knots only with even framings. If we considered framed knots only with odd framings, we obtain the same results as in this section. This suggestion is due to Thang Le.