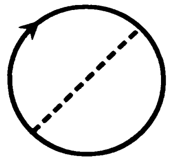

Figure 2.1: The chord diagram Θ

Invariance under RII. The invariance is derived from $R \cdot R^{-1} = R^{-1} \cdot R = 1$.
Invariance under RIII. We obtain the invariance by the hexagon relations (2.6).

□

3 The modified Kontsevich invariant and quantum invariants

In this section we show that quantum invariants recover from the modified Kontsevich invariant; we expect the recovery because of the following historical development from quantum invariants to the modified Kontsevich invariant.

T. Kohno [16] gave an expression of quantum invariants using an iterated integral solution of the Knizhnik-Zamolodchikov equation [15]. Based on Kohno's work, Drinfeld [7] led the universal version of the Knizhnik-Zamolodchikov equation; the solution of it consists of chord diagrams, not depending on a Lie algebra and its representation, and the ordinary solution recovers from the "universal" solution by substituting a Lie algebra and its representation to chord diagrams. After that, Kontsevich gave a definition of an invariant (the Kontsevich invariant) of knots using the universal solution written by the iterated integral. Further J. Murakami and Le [25] gave a combinatorial construction of the invariant (the modified Kontsevich invariant), modifying the invariant for framed links; we denote it by $\hat{Z}(L)$ for an oriented framed link L. Therefore quantum invariants should recover from the modified Kontsevich invariant by substituting a Lie algebra and its representation into chord diagrams.