

1 Preliminaries

1.1 Chord diagrams

In this section we introduce definitions and some properties of chord diagrams.

A *uni-trivalent* graph is a graph, every vertex of which is either univalent or trivalent, where a vertex of a graph is *univalent* (resp. *trivalent*) if there is one edge (resp. there are three edges) of the graph adjacent to the vertex. For a compact oriented 1-dimensional manifold X (possibly with boundary), a *chord diagram* on X is the manifold X together with a uni-trivalent graph whose univalent vertices are on X and whose trivalent vertices are vertex-oriented. Here a trivalent vertex is *vertex-oriented* if a cyclic order of three edges around the vertex is fixed. The *degree* of a chord diagram is half the number of univalent and trivalent vertices of the chord diagram. In figures, we draw X by solid lines and the graph by dashed lines, and each vertex-orientation is fixed in counterclockwise orientation in the plane. For definition of chord diagrams, see also [3, 27].

Definition 1.1. Let X be a compact oriented 1-manifold. We define the vector space $\mathcal{A}(X)$ by

$$\mathcal{A}(X) = \mathbb{C}\{\text{chord diagrams on } X\}/\text{AS, IHX, STU} \quad (1.1)$$

where the AS, IHX and STU relations are shown in Figure 1.1.

Let X be the disjoint union of an interval and a circle. We show an example of a chord diagram below.

