Advanced Studies in Pure Mathematics 86, 2020
Development of Iwasawa Theory — the Centennial of K. Iwasawa's Birth
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My memories of Iwasawa

Ralph Greenberg

Iwasawa 2017 was very meaningful for me. It was a celebration of the 100th anniversary of Iwasawa's birth. It was also a personal anniversary. I first met Iwasawa in the Fall of 1967, just 50 years before. I feel so fortunate that he accepted me as his student. I began to study the theory of Γ-extensions at that time, which we now would call the theory of $\mathbf{Z}_p$-extensions. Actually, Iwasawa adopted that terminology only a few years later. So much has changed in the intervening 50 years. When I was a graduate student, the subject was quite accessible. One could learn virtually everything that was known by studying a relatively small number of papers, mostly Iwasawa's own papers. This is far from true now. Just listening to the diverse lectures at Iwasawa 2017 was a dramatic reminder of how fruitful Iwasawa's ideas have turned out to be and how the subject has really blossomed over the past 50 years. That may be the most important meaning of Iwasawa 2017 for me. It has been a profound privilege for me to witness this blossoming and to be part of it to some extent.

When I was a graduate student, I often met Iwasawa at the afternoon tea in Fine Hall. We would chat briefly and then he would ask if I made any progress on the various questions that he had suggested for me to work on. At some point, I had made some progress on one of the questions. I had managed to find an example where I thought that the λ and μ invariants in his famous formula vanish, but the ν-invariant is nonzero. We went to his office to discuss the example and, with his help, it turned out to work. Iwasawa had actually asked me to look for an example of a totally real number field F and a prime p where the λ-invariant was positive (for what we now call the cyclotomic $\mathbf{Z}_p$-extension of F). I did manage to find quite a variety of examples where both λ and μ vanish and ν is positive, mostly examples where F is a real quadratic field. However, I never succeeded in finding an example where F is totally real and λ (or μ) is positive. There is probably a

Received December 3, 2018.