

Memories of Kenkichi Iwasawa

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In the conference at the University of Tokyo in August 2017 to mark the 100th anniversary of the birth of Kenkichi Iwasawa, the organisers invited me to briefly recall some of my memories of him. The present informal notes are a summary of what I said on that occasion.

When I came to Harvard as a post-doc in September, 1969, I knew nothing of the work of Iwasawa. My Cambridge doctoral thesis had been on proving effective bounds for the solutions of certain diophantine equations by Baker's method in transcendental number theory. However, motivated by the enormous stir in mathematical circles in the two Cambridges and Paris caused by the discovery of the conjecture of Birch and Swinnerton-Dyer, I was keen to start a new direction of research in the general area linking special values of L -functions to exact arithmetic formulae. To my great good fortune, John Tate, who had just returned from a sabbatical in Paris, gave a weekly seminar at Harvard in the fall of 1969 on the conjecture he had recently made with Bryan Birch predicting that, for any totally real number field F , the tame kernel, which we shall denote by R_2F , inside the Milnor K_2F should always be finite, and that its order should be given by a simple formula in terms of the value of the complex zeta function of F at the point $s = -1$. In fact, Tate had visited Cambridge, England, for a few days earlier in the year, and had already then given several very nice lectures on this conjecture. The precise statement of the conjecture, which is now a theorem in view of subsequent work, is that R_2F is finite and that its order is given by

$$(1) \quad \#(R_2F) = w_2(F)\zeta(F, -1),$$

where $\zeta(F, s)$ denotes the complex zeta function of F , and $w_2(F)$ is the largest integer m such that the Galois group of $F(\mu_m)/F$ is annihilated by 2; here μ_m denotes the group of m -th roots of unity. It was known from earlier work of Klingen that $\zeta(F, -1)$ is indeed a rational number. Tate's seminar lectures were a beautiful mixture of conceptual ideas and