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Regularity of minimal and CMC stable hypersurfaces: a brief overview

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§1. Introduction

This article is largely based on the material covered in the author's lecture at the 11th Mathematical Society of Japan Seasonal Institute held in Sapporo in July 2018. Our goal here, as was in the lecture, is to provide a brief discussion of the regularity theory of minimal and constant-mean-curvature (CMC) stable hypersurfaces of Riemannian manifolds, focusing mainly on the recent developments ([BCW], [BelWic-1], [Wic14]). We refer the reader to the articles [Wic14b] and [Wic] for more detailed surveys of the results presented here.

We shall begin by presenting, in Section 2, basic notions and definitions necessary for the statements of the results in later sections. Section 3 contains an overview of the work [Wic14] which provides a regularity and compactness theory for mass bounded stable, stationary (i.e. zero generalized mean curvature) hypersurfaces of a given $(n+1)$-dimensional smooth Riemannian manifold N. Once regularity is known, these hypersurfaces are critical points of the functional $\mathcal{A} : M \mapsto \mathcal{H}^n(M)$ on hypersurfaces $M \subset N$, where $\mathcal{H}^n$ is the n-dimensional Hausdorff measure on N induced by the metric on N. The work [Wic14] can be viewed as providing a complete regularity theory for stable minimal hypersurfaces for the embedded case. It builds on the earlier fundamental work of Schoen–Simon ([SchSim81]) that treated a special case, giving primarily compactness for locally uniformly area bounded stable minimal hypersurfaces with small singular sets. The embeddedness criterion provided by the work [Wic14], namely the absence of *classical singularities* (see the definition in Section 3 below), is readily checked for

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