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Spectral theory on manifolds

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§1. Basic settings

This is a short and slightly simplified report on our article [IS2], in which we studied spectral theory for the Schrödinger operator

$$H = H_0 + V \quad \text{on } M; \quad H_0 = -\frac{1}{2}\Delta = \frac{1}{2}p_i^* g^{ij} p_j, \; p_i = -\mathrm{i}\partial_i.$$

Here (M, g) is a connected Riemannian manifold with Euclidean and/or hyperbolic ends, and V is a real-valued and bounded potential. Self-adjointness of H and H_0 on $\mathcal{H} = L^2(M)$ is realized by the Friedrichs extension. Our main results are Rellich's theorem, the LAP (Limiting Absorption Principle), radiation condition bounds and the Sommerfeld uniqueness result. Proofs depend intensively on commutator arguments with different conjugate operator from the Mourre theory, but we will omit them in this article. Based on the results here, stationary scattering theory and a characterization of asymptotics of generalized eigenfunction are developed in [IS3].

1.1. End structure

Now let us more precisely introduce our basic settings. We formulate and control end structure in a somewhat implicit manner. For examples of manifolds satisfying conditions below we refer to Section 3.

Condition 1. Let (M, g) be a connected Riemannian manifold of dimension $d \geq 1$. There exist $r \in C^\infty(M)$ and $r_0 \geq 2$ such that:

(1) The image of r is $r(M) = [1, \infty)$;

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