Advanced Studies in Pure Mathematics 78, 2018
Singularities in Generic Geometry
pp. 431–448

Implicit Hamiltonian systems and singular curves of distributions

Asahi Tsuchida

§1. Introduction

The subject of the sub-Riemannian geometry is to study a triple $(M, \mathcal{D}, g)$ of a manifold M, a distribution $\mathcal{D}$ on M and a bi-linear positive definite form g on $\mathcal{D}$, which is called a *sub-Riemannian manifold*. Here, a distribution is a sub-bundle of the tangent bundle TM of M. The object appears naturally as a collapsing Riemannian manifold and is a generalization of a Riemannian manifold. However properties of sub-Riemannian manifolds are much different from those of Riemannian manifolds. In fact, not much is known about the properties of exponential maps and even about smoothness of minimizers. For example there is a problem which is open for decades: "Are all locally minimizers smooth on any sub-Riemannian manifold?"

In this article we give a new clue to study of minimizers on sub-Riemannian manifolds.

For a distribution $\mathcal{D}$, there is an important class of curves called horizontal curves. A *horizontal curve* is an absolutely continuous curve $\gamma\colon I \to M$ such that $\dot{\gamma}(t)$ is a measurable and bounded map which satisfies $\dot{\gamma}(t) \in \mathcal{D}_{\gamma(t)}$ for almost every $t \in I$.

According to Chow–Rashevsky's theorem, if a distribution $\mathcal{D}$ on a connected manifold M satisfies Hörmander's condition, every two points are connected by a horizontal curve. For a sub-Riemannian manifold

Received July 9, 2016.
Revised January 28, 2017.
2010 *Mathematics Subject Classification.* Primary 53C17; Secondary 70F25, 70H45.