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Singularities of frontals

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§1. Introduction

In this survey article we introduce the notion of frontals, which provides a class of generalised submanifolds with singularities but with well-defined tangent spaces. We present a review of basic theory and known studies on frontals in several geometric problems from singularity theory viewpoints. In particular, in this paper, we try to give some of detailed proofs and related ideas, which were omitted in the original papers, to the basic and important results related to frontals.

We start with one of theoretical motivations for our notion "frontal". Let M be a C^∞ manifold of dimension m, which is regarded as an ambient space. Suppose $n \leq m$ and let $f : N \to M$ be an *immersion* of an n-dimensional C^∞ manifold N, which is regarded as a parameter space, to M. Then for each point $t \in N$, we have the n-plane $f_*(T_t N)$, the image of the differential map $f_* : T_t N \to T_{f(t)} M$ at t in the tangent space $T_{f(t)} M$. Thus we have a field of tangential n-planes $\{f_*(T_t N)\}_{t \in N}$ along the immersion f. Moreover if M is endowed with a Riemannian metric, then we have also a field of tangential $(m-n)$-planes $f_*(T_t N)^\perp$ along f. Taking those vector bundles we can develop differential topology, theory of characteristic classes and so on of immersed submanifolds. Besides, taking local adapted frames for immersions, we can develop differential geometry of immersed submanifolds in terms of frames. Then a natural and challenging problem arises to us on the possibility to find a natural class of singular mappings enjoying the same properties as immersed submanifolds and to develop generalised topological and geometric theories on them.

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