

Locally nilpotent derivations of rings graded by an abelian group

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§1. Introduction

Among the most important tools in the study of locally nilpotent derivations (LNDs) of commutative rings are its \mathbb{Z} -gradings and the homogeneous derivations associated to them. Gradings which involve other totally ordered abelian groups have also been used to study LNDs, though to a lesser extent. Gradings which involve non-totally ordered abelian groups have been largely ignored in this context, since it is no longer possible to associate a highest-degree homogeneous derivation to a given derivation in this case. However, it turns out that one can still get valuable information about LNDs from such gradings. In [7], the second and third authors studied LNDs of certain rings graded by a finite cyclic group. Their results were applied to show that some families of Pham-Brieskorn threefolds are rigid, i.e., their coordinate rings have no nonzero LNDs.

In the present work, we generalize the theory developed in that paper to the case of rings graded by arbitrary abelian groups. Let B be a domain of characteristic zero graded by an abelian group G . For any subgroup H of G , let B_H be the subring of B generated by the nonzero homogeneous elements of B whose degrees belong to H . An element x of B is G -critical if it is homogeneous and nonzero, and if there exists a

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