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Fundamental groups of symplectic singularities

Yoshinori Namikawa

§1. Introduction

Let (X, ω) be an affine symplectic variety. By definition (cf. [2]), X is an affine normal variety and ω is a holomorphic symplectic 2-form on the regular locus X_{reg} of X such that it extends to a holomorphic (not necessarily symplectic) 2-form on a resolution $\tilde{X}$ of X. In this article we also assume that X has a $\mathbf{C}^*$-action with positive weights and that ω is homogeneous with respect to the $\mathbf{C}^*$-action. More precisely, the affine ring R of X is positively graded: $R = \oplus_{i \geq 0} R_i$ with $R_0 = \mathbf{C}$ and there is an integer l such that $t^*\omega = t^l \cdot \omega$ for all $t \in \mathbf{C}^*$. Since X has canonical singularities, we have $l > 0$ ([8], Lemma (2.2)). Affine symplectic varieties are constructed in various ways such as nilpotent orbit closures of a semisimple complex Lie algebra (cf. [4]), Slodowy slices to nilpotent orbits ([9]) or symplectic reductions of holomorphic symplectic manifolds with Hamiltonian actions. These varieties come up with $\mathbf{C}^*$-actions and the above assumption of the $\mathbf{C}^*$-action is satisfied in all examples we know.

In the previous article [8] we posed a question:

Problem. *Is the fundamental group $\pi_1(X_{reg})$ finite ?*

Such fundamental groups are explicitly calculated by a group-theoretic method when X is a nilpotent orbit closure (cf. [4]). However no general results are known.

In this short note we give a partial answer to this question. Namely we have

Theorem 1.1. *The algebraic fundamental group $\hat{\pi}_1(X_{reg})$ is a finite group.*

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