

## Brownian motion on foliated complex surfaces, Lyapunov exponents and applications

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### § Introduction

These lectures are motivated by the dynamical study of differential equations in the complex domain. Most of the topic will concern *holomorphic foliations on complex surfaces*, and their connections with the theory of *complex projective structures on curves*. In foliation theory, the interplay between geometry and dynamics is what makes the beauty of the subject. In these lectures, we will try to develop this relationship even more.

On the geometrical side, we have generalizations of the *foliation cycles* introduced by Sullivan, see [68]: namely the *foliated harmonic currents*, see e.g. [36, 4]. Those currents permit to think of the foliation as if it were a genuine algebraic curve. For instance, one can associate a homology class, compute intersections with divisors on the surface etc. These currents can often be viewed as limits of the (conveniently normalized) currents of integration on large leafwise domains defined via the uniformization of the leaves. This point of view, closely related to Nevanlinna theory, is very fruitful in the applications as we will see. See [5, 28].

On the dynamical side, the *leafwise Brownian motions* (w.r.t. to some hermitian metric on the tangent bundle to the foliation, e.g. coming from uniformization of leaves) generate a Markov process on the

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