On ℚ-Fano 3-folds of Fano index 2

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To Shigefumi Mori in friendship and admiration

§1.   Introduction

1.1.   ℚ-Fano 3-folds

A ℚ-*Fano* 3-*fold* is a projective 3-fold X with at worst terminal singularities and ample anticanonical divisor $-K_X$. Here, bearing in mind Mori's fundamental notion of extremal ray, we assume also that X is ℚ-factorial and has rank 1, that is, $\operatorname{Pic} X \simeq \mathbb{Z}$ or equivalently, $\operatorname{Cl} X \otimes \mathbb{Q} \simeq \mathbb{Q}$. We define the *Fano* and ℚ-*Fano index* of X by:

$$\mathrm{q_F}(X) := \max\{q \in \mathbb{Z} \mid -K_X \sim qA \text{ with } A \text{ a Weil divisor}\},$$

$$\mathrm{q_\mathbb{Q}}(X) := \max\{q \in \mathbb{Z} \mid -K_X \sim_\mathbb{Q} qA \text{ with } A \text{ a Weil divisor}\},$$

where $\sim$ is linear equivalence and $\sim_\mathbb{Q}$ is ℚ-linear equivalence. Clearly, $\mathrm{q_F}(X)$ divides $\mathrm{q_\mathbb{Q}}(X)$, and the two coincide unless $K_X + qA \in \operatorname{Cl} X$ is a nontrivial torsion element. An important invariant of a ℚ-Fano 3-fold is its *genus* $\mathrm{g}(X) := \dim |-K_X| - 1$.

1.2.   Background facts

Kaori Suzuki [Suz04] restricts the ℚ-Fano index of X to one of

$$(1.2.1) \qquad \mathrm{q_\mathbb{Q}}(X) \in \{1, \dots, 11, 13, 17, 19\}.$$

See also [Pro10b, Lemma 3.3]. Moreover, the following results are due to the first author.