

On \mathbb{Q} -Fano 3-folds of Fano index 2

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To Shigefumi Mori in friendship and admiration

§1. Introduction

1.1. \mathbb{Q} -Fano 3-folds

A \mathbb{Q} -Fano 3-fold is a projective 3-fold X with at worst terminal singularities and ample anticanonical divisor $-K_X$. Here, bearing in mind Mori's fundamental notion of extremal ray, we assume also that X is \mathbb{Q} -factorial and has rank 1, that is, $\text{Pic } X \simeq \mathbb{Z}$ or equivalently, $\text{Cl } X \otimes \mathbb{Q} \simeq \mathbb{Q}$. We define the *Fano* and \mathbb{Q} -Fano index of X by:

$$q_{\mathbb{F}}(X) := \max\{q \in \mathbb{Z} \mid -K_X \sim qA \text{ with } A \text{ a Weil divisor}\},$$

$$q_{\mathbb{Q}}(X) := \max\{q \in \mathbb{Z} \mid -K_X \sim_{\mathbb{Q}} qA \text{ with } A \text{ a Weil divisor}\},$$

where \sim is linear equivalence and $\sim_{\mathbb{Q}}$ is \mathbb{Q} -linear equivalence. Clearly, $q_{\mathbb{F}}(X)$ divides $q_{\mathbb{Q}}(X)$, and the two coincide unless $K_X + qA \in \text{Cl } X$ is a nontrivial torsion element. An important invariant of a \mathbb{Q} -Fano 3-fold is its *genus* $g(X) := \dim |-K_X| - 1$.

1.2. Background facts

Kaori Suzuki [Suz04] restricts the \mathbb{Q} -Fano index of X to one of

$$(1.2.1) \quad q_{\mathbb{Q}}(X) \in \{1, \dots, 11, 13, 17, 19\}.$$

See also [Pro10b, Lemma 3.3]. Moreover, the following results are due to the first author.

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