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Restricted Lazarsfeld–Mukai bundles and canonical curves

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Dedicated to Professor Shigeru Mukai on his sixtieth birthday, with admiration

For a $K3$ surface S, a smooth curve $C \subset S$ and a globally generated linear series $A \in W^r_d(C)$ with $h^0(C, A) = r + 1$, the *Lazarsfeld-Mukai* vector bundle $E_{C,A}$ is defined via the following elementary modification on S

$$0 \longrightarrow E^{\vee}_{C,A} \longrightarrow H^0(C, A) \otimes \mathcal{O}_S \longrightarrow A \longrightarrow 0. \tag{1}$$

The bundles $E_{C,A}$ have been introduced more or less simultaneously in the 80's by Lazarsfeld [L1] and Mukai [M1] and have acquired quite some prominence in algebraic geometry. On one hand, they have been used to show that curves on general $K3$ surfaces verify the Brill-Noether theorem [L1], and this is still the only class of smooth curves known to be general in the sense of Brill-Noether theory in every genus. When $\rho(g, r, d) = 0$, the vector bundle $E_{C,A}$ is rigid and plays a key role in the classification of Fano varieties of coindex 3. For $g = 7, 8, 9$, the corresponding Lazarsfeld-Mukai bundle has been used to coordinatize the moduli space of curves of genus g , thus giving rise to a new and more concrete model of $\mathcal{M}_g$, see [M2], [M3], [M4]. Furthermore, Lazarsfeld-Mukai bundles of rank two have led to a characterization of the locus in $\mathcal{M}_g$ of curves lying on $K3$ surfaces in terms of existence of linear series with unexpected syzygies [F], [V]. For a recent survey on this circle of ideas, see [A].

Recently, Lazarsfeld-Mukai bundles have proven to be effective in shedding some light on an interesting conjecture of Mercat in Brill-Noether theory, see [FO1], [FO2], [LMN]. Recall that the Clifford index

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