More lectures on Hilbert schemes of points on surfaces

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**Dedicated to Professor Shigeru Mukai on the occasion of his
60th birthday**

§ Introduction

This paper is based on author's lectures at Kyoto University in 2010
Summer, and in the 6th MSJ-SI 'Development of Moduli Theory' at
RIMS in June 2013.

The purpose of lectures was to review several results on Hilbert
schemes of points which were obtained after author's lecture note [24]
was written. Among many results, we choose those which are about
equivariant homology groups $H_*^T(X^{[n]})$ of Hilbert schemes of points on
the affine plane $X = \mathbb{C}^2$ with respect to the torus action. Study of equi-
variant homology groups increases its importance recently. In particular,
it is a basis of the AGT correspondence between instanton moduli spaces
on $\mathbb{C}^2$ and the representation theory of W-algebras, which is a very hot
topic now (see e.g., [20]).

We omit proofs if they are present in [24], but give self-contained
proofs otherwise. In this sense, this should be read after [24].

The paper is organized as follows. In §1, we review basics on equi-
variant (co)homology groups. It will be basis of subsequent sections. In
§2 we construct the Fock representation of the Heisenberg algebra on
$\bigoplus H_*^T(X^{[n]})$, following [24, Ch. 8] as well as an idea of Vasserot [27].
In §3 we explain a geometric realization of Jack symmetric functions
as fixed point classes in $H_*^T(X^{[n]})$ by Li-Qin-Wang [17]. We also give
author's unpublished result, which was used in [17]. As applications,
we give geometric proofs of the norm formula and Pieri formula of Jack