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Resonance varieties of arrangement complements

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§1. Introduction

This paper is a survey based on the lecture given by the author in the conference "Arrangements of Hyperplanes -Sapporo, 2009" at the University of Hokkaido. The main topic of the survey is the resonance varieties of the complement of an arrangement of several linear complex hyperplanes. These varieties can be defined for an arbitrary topological space M as the jumping loci for the kind of 'secondary cohomology' of M. More precisely this is the cohomology of the graded commutative algebra $H^*(M)$ provided with the differential given by the multiplication by an element of $H^1(M)$. This cohomology has appeared first in topology as the first sheet of the Farber–Novikov spectral system (see, for example [25]) which converges to cohomology with local coefficients for compact manifolds. For arrangement complements first results for this cohomology were vanishing theorems from [30] and [35] and comparison theorems [14]. The jumping loci for this cohomology were first considered explicitly in [15] and called resonance varieties. In [1], this cohomology was considered for an arbitrary graded commutative algebra as the measure of its complexity. For initial results about resonance varieties over arbitrary fields see [16].

At the beginning, the resonance varieties were mainly considered due to their connections with the jumping loci for the cohomology with local coefficients; the most recent results about these connections can be found in [10, 11]. Now resonance varieties appear in many areas of arrangement theory. The most recent appearance is in [5, 6] where these varieties have been used for results on the Milnor fiber cohomology and roots of b-functions. There are also several recent papers (see, for example

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