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Perverse coherent sheaves on blow-up. I. A quiver description

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§ Introduction

This is the first of three papers studying moduli spaces of a certain class of coherent sheaves, which we call *stable perverse coherent sheaves*, on the blowup of a projective surface. They are used to relate usual moduli spaces of stable sheaves on a surface and its blowup.

Let us give the definition for general case though we will consider only framed sheaves on the blowup $\widehat{\mathbb{P}}^2$ of the projective plane $\mathbb{P}^2$ in this paper. Let $p\colon \widehat{X} \to X$ be the blowup of a smooth projective surface X at a point. Let C be the exceptional divisor. A *stable perverse coherent sheaf* E on $\widehat{X}$, with respect to an ample line bundle H on X, is

(1) E is a coherent sheaf on $\widehat{X}$,
(2) $\operatorname{Hom}(E(-C), \mathcal{O}_C) = 0$,
(3) $\operatorname{Hom}(\mathcal{O}_C, E) = 0$,
(4) p_*E is slope stable with respect to H.

We will consider the moduli spaces of coherent sheaves E on $\widehat{X}$ such that $E(-mC)$ is stable perverse coherent for $m \geq 0$. This depends on m.

Let us first explain how we find the definition of stable perverse coherent sheaves. Our definition comes from two sources. The first one is Bridgeland's paper [1], from which we take the name "perverse coherent" sheaves. He introduced a new t-structure on the derived category $\mathrm{D}^b(\operatorname{Coh} Y)$ of coherent sheaves on Y for a birational morphism $f\colon Y \to X$ such that (1) $\mathbf{R}f_*(\mathcal{O}_Y) = \mathcal{O}_X$ and (2) $\dim f^{-1}(x) \leq 1$ for any $x \in X$. An object $E \in \mathrm{D}^b(\operatorname{Coh} Y)$ is called *perverse coherent* if it is in the heart $\operatorname{Per}(Y/X)$ of the t-structure. Then he considered the moduli spaces of perverse ideal sheaves which are subobjects of the structure

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