Advanced Studies in Pure Mathematics 58, 2010
Algebraic and Arithmetic Structures of Moduli Spaces (Sapporo 2007)
pp. 159–171

Some questions about $\mathcal{G}$-bundles on curves

Georgios Pappas* **and Michael Rapoport**

§1.

The purpose of this note is to discuss the geometry of moduli stacks of various types of bundles over a curve. We suggest that the main elements of the theory of moduli of G-bundles for a constant reductive algebraic group G as developed by Beauville, Laszlo, Faltings and other authors should extend to a theory of moduli of $\mathcal{G}$-torsors for a large class of algebraic group schemes $\mathcal{G}$ which are not necessarily constant over the curve. The class we consider is that of smooth group schemes over the curve with reductive generic fiber which have the property that each place of the curve the completion of the group scheme is a "parahoric group scheme" of the type constructed by Bruhat–Tits. In addition to the classical case above, the corresponding moduli stacks include the moduli of parabolic G-bundles but also other interesting examples such as the moduli of Prym line bundles (Prym varieties) or moduli of bundles together with (not always perfect) symplectic, orthogonal or hermitian pairings. Our approach uses the theory of loop groups.

In [PR], we introduced and studied the loop group attached to a linear algebraic group over a Laurent series field $k((t))$ where k is an algebraically closed field. To a (connected) reductive algebraic group H over $k((t))$ there is associated the ind-group scheme LH over k, with points with values in a k-algebra R equal to $H(R((t)))$. If P is a parahoric subgroup of $H(k((t)))$, Bruhat and Tits have associated to P a smooth group scheme with connected fibers over Spec $(k[[t]])$, with generic fiber H and with group of $k[[t]]$-rational points equal to P. Denoting by the same symbol P this group scheme, there is associated to it a group scheme L^+P over k, with points with values in a k-algebra R equal to $P(R[[t]])$. The $fpqc$-quotient $\mathcal{F}_P = LH/L^+P$ is representable by an

Received March 31, 2009.
Revised June 3, 2009.
* Partially supported by NSF grant DMS08-02686.