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On the contact structure of a class of real analytic germs of the form $f\bar{g}$

Dedicated to Professor Mutsuo Oka on the occasion of his 60th birthday

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§1. Introduction

Let $(h, O) : (\mathbb{C}^{n+1}, O) \to (\mathbb{C}, 0)$ be a holomorphic germ with $h(O) = 0$, where O is the origin of $\mathbb{C}^{n+1}$. The intersection L_h of $h^{-1}(0)$ with a sphere S^{2n+1}_ε centered at $O \in \mathbb{C}^{n+1}$ with sufficiently small radius $\varepsilon > 0$ is called the *link* of (h, O). In [15], J. Milnor proved that the argument map $h/|h| : S^{2n+1}_\varepsilon \setminus L_h \to S^1$ is a locally trivial fibration and that, under a certain condition, a real analytic germ also defines a locally trivial fibration over S^1. There are several studies concerning this condition, see for instance [25, Ch. VII and VIII] and the references therein.

In [19], A. Pichon studied real analytic germs of the form $(f\bar{g}, O)$, where (f, O) and (g, O) are holomorphic germs from $(\mathbb{C}^2, O)$ to $(\mathbb{C}, 0)$ with isolated singularities and with no common branches. Here $\bar{g}$ represents the conjugation of g. In particular, a condition for the link $L_{f\bar{g}}$ to be fibred is given in terms of the multiplicities on resolution graphs of (f, O) and (g, O). Then she and J. Seade proved in [20] that $f\bar{g}/|f\bar{g}| : S^3_\varepsilon \setminus L_{f\bar{g}} \to S^1$ is a locally trivial fibration if and only if $(f\bar{g}, O)$ satisfies the fibrability condition in [19] in more general context. In [3], A. Bodin and Pichon studied the multilinks of meromorphic functions of the form f/g and represented the fibrability condition for $(f\bar{g}, O)$ in terms of special fibres of f/g.

Let M be an oriented, closed, smooth 3-manifold. A fibration from a link complement of M to S^1 is called an *open book decomposition* of

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