Advanced Studies in Pure Mathematics 55, 2009
Noncommutativity and Singularities
pp. 269–279

Flop invariance of the topological vertex

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§1. Introduction

This is a summary of [6] which is a joint work with Yukiko Konishi. A toric Calabi–Yau (CY) threefold is a non-singular toric threefold with trivial canonical bundle[1] . The topological vertex is an algorithm which enables us to write down an explicit formula for the generating function of all genus Gromov–Witten (GW) invariants of toric CY threefolds. The formula takes a combinatorial form and is written in terms of skew-Schur functions. This method was developed in [1] based on the geometric transitions and the duality to the Chern–Simons theory. A mathematical theory (including a rigorous definition of GW invariants for toric CY threefolds) has been developed later in [8].

It was discovered [2, 13] that there is a kind of analytic continuation process on Kähler cones which links string theories on birationally equivalent CY manifolds. Motivated by these works, the transformation property of GW invariants of projective CY threefolds under flops was studied in [7] (see also [9]). In [6], the same problem was studied for general toric CY threefolds using the topological vertex. Some special cases were studied earlier in [5].

Now we state our main result. Let X be a toric CY threefold and $N_{g,\beta}(X) \in \mathbb{Q}$ be the GW invariant of X with the genus $g \geq 0$ and the degree $\beta \in H_2(X, \mathbb{Z})$[2].

Received August 2, 2007.
Supported by 21st Century COE Program at Department of Mathematics, Hokkaido University.

[1]Varieties which satisfy both the toric condition and the CY condition are necessary non-compact. They are even not quasi-projective in general.

[2]It is defined by the virtual counting of stable maps into X from the genus g curves and the prescribed homology class β of the images. See [8] for a precise definition of GW invariants of toric CY threefolds.