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Partial regularity and its application to the blow-up asymptotics of parabolic systems modelling chemotaxis with porous medium diffusion

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§1. Introduction

We consider the following reaction-diffusion equation:

$$
(\mathrm{KS})_m \begin{cases} \partial_t u = \Delta u^m - \nabla \cdot \left(u^{q-1} \nabla v\right), & x \in \mathbb{R}^N,\ t > 0, \\ 0 = \Delta v - \gamma v + u, & x \in \mathbb{R}^N,\ t > 0, \\ u(x,0) = u_0(x), & x \in \mathbb{R}^N. \end{cases}
$$

Throughout this article, we assume that $N \geq 3$, and that m, q, and γ are the constants satisfying

$$m > 1,\ q \geq 2,\ \gamma > 0.$$

The initial data u_0 is a non-negative function satisfying

$$u_0 \in L^1 \cap L^\infty(\mathbb{R}^N) \quad \text{with } u_0^m \in H^1(\mathbb{R}^N).$$

This equation is often called the Keller–Segel model describing the motion of the chemotaxis molds, where $u(x,t)$ and $v(x,t)$ denote the density of amoebae and the concentration of the chemo-attractant, respectively. (we refer to Keller–Segel [6], Horstman [4], Suzuki [17].)

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