Masaki Kashiwara and representation theory

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It is a great honor for me to have an occasion to contribute an article on the work of Masaki Kashiwara in representation theory.

Let me start with my reminiscence of the period when I first met him. My main interest when I started my career as a mathematician around 1980 was on the infinite dimensional highest weight representations of semisimple Lie algebras. At that period the most important problem in this area was to determine the characters of irreducible highest weight modules. A precise conjecture had just been formulated by Kazhdan and Lusztig [36]. One day in 1981 I heard the news that the problem was solved by Brylinski–Kashiwara [5] and Beilinson–Bernstein [4], and I obtained a preprint of [5]. I was then a pure algebraist having no idea about D-modules, so I could not understand the language used in the preprint at all. But I gradually learned the theory of D-modules since then. Around 1982 I caught an idea that a problem in representation theory can be reformulated in the framework of D-modules, and wrote a letter to Kashiwara. Fortunately he was interested in the problem and it grew into my first work with him [26]. I still remember vividly my enthusiasm of the day when I got an unexpected phone call from the great mathematician mentioning his interest in the problem I posed.

A friend of mine once told me that the most important thing in achieving a success in collaboration with Kashiwara is to make him interested in the problem. I agree with this from my experiences including the above mentioned first one.

In this short manuscript I will describe some of the works of Kashiwara in the field of representation theory.

§1. Soliton equation

Methods from representation theory often shed light on problems in the theory of integrable models in mathematical physics. Conversely,

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