Masaki Kashiwara and algebraic analysis

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It is a great honor[1] to present here some aspects of the work of Masaki Kashiwara.

Recall that Masaki's work covers many fields of mathematics, algebraic and microlocal analysis of course, but also representation theory, Hodge theory, integrable systems, quantum groups and so on. Also recall that Masaki had many collaborators, among whom Daniel Barlet, Jean–Luc Brylinski, Etsuro Date, Ryoshi Hotta, Michio Jimbo, Seok–Jin Kang, Takahiro Kawai, Tetsuji Miwa, Kiyosato Okamoto, Toshio Oshima, Mikio Sato, myself, Toshiyuki Tanisaki and Michèle Vergne.

In each of the domain he approached, Masaki has given essential contributions and made important discoveries, such as, for example, the existence of crystal bases in quantum groups. But in this talk, I will restrict myself to describe some part of his work related to microlocal and algebraic analysis.

The story begins long ago, in the early sixties, when Mikio Sato created a new branch of mathematics now called "Algebraic Analysis". In 1959/60, M. Sato published two papers on hyperfunction theory [24] and then developed his vision of analysis and linear partial differential equations in a series of lectures at Tokyo University (see [1]). If M is a real analytic manifold and X is a complexification of M, hyperfunctions on M are cohomology classes supported by M of the sheaf $\mathcal{O}_X$ of holomorphic functions on X. It is difficult to realize now how Sato's point of view was revolutionary at that time. Sato's hyperfunctions are constructed using tools from sheaf theory and complex analysis, when people were totally addicted to functional analysis, and when the separation between real and complex analysis was very strong.

Then came Kashiwara's thesis, dated December 1970 (of course written in Japanese, but translated in English and published by the French