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Asymptotic geometry of foliations and pseudo-Anosov flows — a survey

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This expository article describes certain aspects of large scale geometry of foliations and flows in 3-manifolds with Gromov hyperbolic fundamental group. The first part is a survey of ideas and previous results about asymptotic behavior of leaves of foliations and flow lines. It reviews classical ideas such as quasi-isometries, quasigeodesics and ideal boundaries. This part includes a description of the seminal work of Cannon-Thurston on fibrations and suspension pseudo-Anosov flows as well as other important results concerning certain classes of foliations. The second part of the article describes more recent topics: What information can be obtained about the asymptotic structure of the universal cover of a manifold using only the dynamics of a pseudo-Anosov flow in the manifold? We explain how to create a dynamical systems ideal boundary for a certain class of such flows and a corresponding compactification of the universal cover. We then explain how these objects are strongly related to the large scale geometry of the manifolds, the large scale geometry of the flows themselves and also of some classes of foliations. Details and proofs of the results in the second part are found in [Fe8].

Let $\mathcal{F}$ be a Reebless foliation in M^3 with Gromov hyperbolic fundamental group, which is not virtually $\mathbf{Z}$. The lifted foliation to the universal cover $\widetilde{M}$ will be denoted by $\widetilde{\mathcal{F}}$. By Novikov's fundamental theorem, the leaves of $\mathcal{F}$ are incompressible and lift to simply connected leaves in $\widetilde{M}$ [No]. The leaves in $\widetilde{\mathcal{F}}$ cannot be spheres or else $\widetilde{M}$ would

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