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Period problems for mean curvature one surfaces in H^3 (with applications to surfaces of low total curvature)

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§1. Introduction

There is a wide body of knowledge about minimal surfaces in Euclidean 3-space $\boldsymbol{R}^3$, and there is a canonical local isometric correspondence (sometimes called the Lawson correspondence) between minimal surfaces in $\boldsymbol{R}^3$ and CMC-1 (constant mean curvature one) surfaces in hyperbolic 3-space H^3 (the complete simply-connected 3-manifold of constant sectional curvature -1). This has naturally led to the recent interest in and development of CMC-1 surfaces in H^3 in the last decade. There are now many known examples, and it is a natural next step to classify all such surfaces with low total absolute curvature.

By this canonical local isometric correspondence, minimal immersions in $\boldsymbol{R}^3$ are locally equivalent to CMC-1 immersions in H^3. But there are interesting differences between these two types of immersions on the global level. There are period problems on non-simply-connected domains of the immersions, which might be solved for one type of immersion but not the other. Solvability of the period problems is usually more likely in the H^3 case, leading to a wider variety of surfaces there. For example, a genus 1 surface with finite total curvature and two embedded ends cannot exist as a minimal surface in $\boldsymbol{R}^3$, but it does exist as a CMC-1 surface in H^3 [RS]. And a genus 0 surface with finite total curvature and two embedded ends exists as a minimal surface in $\boldsymbol{R}^3$ only if it is a surface of revolution, but it may exist as a CMC-1 surface in

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