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Kähler Ricci solitons

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§1. Introduction

We give a survey of Ricci solitons in a Kähler background. The emphasis is on joint work with Christina Tønnesen–Friedman and Galliano Valent [11].

Let (M, J) be a complex manifold. Consider pairs (g, V) consisting of a Kähler metric g and a real holomorphic vector field V on M, such that JV is an isometry of g and

$$\rho - \lambda\Omega = L_V\Omega, \tag{1}$$

where ρ is the Ricci form, Ω is the Kähler form and λ is a constant. Such structures are called *quasi-Einstein Kähler metrics* or *Kähler Ricci solitons* [4, 5, 7, 8, 12].

Remark 1. Quasi-Einstein metrics are solitons for the Hamilton flow [8]

$$\frac{d}{dt}g_t = -r_t + \frac{\overline{s_t}}{n}g_t, \tag{2}$$

where r_t is the Ricci curvature tensor and $\overline{s_t}$ is the average scalar curvature of g_t. Indeed, if g_0 is quasi-Einstein then $(\Phi_{-t})^*g_0$ solves (2), where $\Phi_t = \exp(tV)$. Thus if g_0 is quasi-Einstein but not Einstein, then g_t does not converge to an Einstein metric – it flows along V as a soliton.

Remark 2. Friedan [6] studied quasi-Einstein metrics in connection with bosonic σ-models. He showed that the one-loop renormalizability

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