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Darboux transformations and generalized self-dual Yang-Mills flows

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§1. Introduction

The self-dual Yang-Mills equations are important partial differential equations in mathematical physics and have significant applications in mathematics [2, 3]. Besides the outstanding results related to 4-dimensional topology [5], it was known long ago that the self-dual Yang-Mills equations are an integrable system in the sense that they are the integrability condition of an overdetermined system of partial differential equations with a spectral parameter λ [12, 16]. Consequently, the rapidly developing theory of integrable systems can be applied to the self-dual Yang-Mills equations. Moreover, it has been found that many known soliton equations are reductions of the self-dual Yang-Mills equations [1, 17].

The self-dual Yang-Mills equations are equations in 4 dimensional space. It is interesting to generalize the equations to higher dimensions [13, 14, 18]. In [14], the generalization to $\mathbf{R}^{4n}$ is a simple case. In [8], more general integrable systems called generalized self-dual Yang-Mills flows (GSYMF) in $\mathbf{R}^{2n,1}$ were introduced. These are dynamical systems on the moduli space of the generalized self-dual Yang-Mills equations. The systems are very general in the sense that almost all known soliton equations of all dimensions are the reduction of these systems. It has been found that the Darboux transformation method is applicable to the GSYMF, giving new solutions explicitly. For the multi-soliton solutions, the phenomena of separation of solitons and confinement of solitons occurs.

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