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Timelike surfaces with harmonic inverse mean curvature

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Introduction

In this paper we introduce the notion of *timelike surfaces with harmonic inverse mean curvature* in 3-dimensional Lorentzian space forms, and study their fundamental properties.

In classical differential geometry, surfaces of constant mean curvature (CMC surfaces) have been studied extensively [1]. As a generalization of CMC surfaces, Bobenko [2] introduced the notion of *surface with harmonic inverse mean curvature* (HIMC surface). He showed that HIMC surfaces admit a Lax representation with *variable* spectral parameter. In [5], Bobenko, Eitner and Kitaev showed that the Gauss equations of θ-isothermic HIMC surfaces reduce to the ordinary differential equation:

$$(*) \qquad \left(\frac{q''(t)}{q'(t)}\right)' - q'(t) = \mathcal{S}(t)\left(2 - \frac{q^2(t)+c}{q'(t)}\right), \quad q'(t) < 0,$$

with $c = \theta^2 > 0$. Here the coefficient function $\mathcal{S}(t)$ is $1/\sin^2(2t)$, $1/\sinh^2(2t)$ or $1/t^2$. This ordinary differential equation is called the *generalized Hazzidakis equation*. Bobenko, Eitner and Kitaev [5] solved $(*)$ in terms of Painlevé transcendents P_{V} and P_{VI}.

For $c < 0$, solutions to $(*)$ do not describe surfaces in Euclidean 3-space. It seems to be interesting to find "corresponding surfaces" to such solutions.

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