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On the Castelnuovo-Severi inequality for a double covering

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§0. Introduction, motivation and the results

Let C be a smooth projective irreducible complex algebraic curve of genus $g \geq 2$. We denote g^1_d by a 1-dimensional possibly incomplete linear system of degree d on C. For any $d \geq g+1$, every curve C of genus g has a base point free g^1_d which may be taken as a general pencil of a general element in $W^{d-g}_d(C) = J(C)$. If C is a hyperelliptic curve with the hyperelliptic pencil g^1_2, it is well-known that any base point free pencil of degree $d \leq g$ is a subsystem of the complete rg^1_2 where $r = \frac{d}{2}$; cf. [1, p.109]. In particular, the only base point free and complete pencil on a hyperelliptic curve is the g^1_2. On the other hand, a non-hyperelliptic curve C has a base point free and complete pencil of degree g, by taking off $g-2$ general points from the very ample canonical linear system $|K_C|$.

Furthermore, a theorem of Harris asserts that any non-hyperelliptic curve of genus g has a base point free and complete pencil of degree $g-1$; cf. [1, p.372]. However, this seemingly simple fact requires a proof which is somewhat involved. Especially, in case C is a bi-elliptic curve, one needs to show that the variety $W^1_{g-1}(C)$ consisting of special pencils of degree $g-1$ is reducible by using enumerative methods; see also [3, Proposition 3.3],[6, Proposition 2.5] for the other proofs concerning the existence of a base point free and complete pencil g^1_{g-1} on a bi-elliptic curve. At this point, it is worthwhile to recall the following classical result known as Castelnuovo-Severi inequality.

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