

Towards a categorical construction of Lie algebras

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*To the memory of Nguyen Huu Duc
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Preface

This is an introduction to the program which we call “towards a categorical construction of Lie Algebras”. That is, from the data of a system of 4 integers $W := (a, b, c; h)$, called a *regular system of weights*, satisfying an arithmetic condition, we want to construct a certain generalization \mathfrak{g}_W of a simple Lie algebra. Precisely, to a weight system, we first associate a surface with a singular point. Then, using the geometry of the singularity, a triangulated category is attached. Finally, we want to read Lie theoretic data from the category and to construct the algebra \mathfrak{g}_W .¹ The program is still in its early stages, and, in the present paper, we are mainly concerned with some categorical aspects of the program, and then ask questions on the possible constructions of Lie algebras.

The organization of the paper is as follows. In §1-9, we start by recalling the classical relations of simple or simply elliptic singularities with simple or elliptic Lie algebras, respectively, as the prototype of relations between singularities and Lie algebras. This part is rather

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¹This is a part of the long program “a categorical construction of primitive forms” (see [Mat][Od1][Sa7] and Footnote 11 for a definition of a primitive form, and consult the overview articles [Sa15] and [Sa19]). We expect that a good class of primitive forms are constructed from the Lie algebra \mathfrak{g}_W associated with regular systems of weights W (see §4 and 12). In the present paper, we are concerned with the part of the program before the construction of the Lie algebra, and most parts are readable without a knowledge of a primitive form.