Advanced Studies in Pure Mathematics 50, 2008
Algebraic Geometry in East Asia — Hanoi 2005
pp. 1–100

# A survey on Zariski pairs

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***Dedicated to Professor Kenji Ueno on his sixtieth birthday.***

## § Introduction

In 1929, O. Zariski published a paper entitled *"On the Problem of Existence of Algebraic Functions of Two Variables Possessing a Given Branch Curve"* [130] where the following question was considered:

*Does an algebraic function $z$ of $x$ and $y$ exist, possessing a preassigned curve $f$ as branch curve?*

As Zariski pointed out in the *Introduction* of [130], this question was first considered by Enriques and the problem is reduced to finding the fundamental group of the complement of the given curve (the word *complement* is understood and often omitted for short). Zariski considered some explicit cases and *proved* important results. Here we detail some of the most relevant:

(Z1) If two curves lie in a connected family of equisingular curves, then they have isomorphic fundamental groups.

(Z2) If a continuous family $\{\mathcal{C}_t\}_{t\in[0,1]}$ is equisingular for $t \in (0,1]$ and $\mathcal{C}_0$ is reduced, then there is a natural epimorphism $\pi_1(\mathbb{P}^2 \setminus \mathcal{C}_0, p_0) \twoheadrightarrow \pi_1(\mathbb{P}^2 \setminus \mathcal{C}_t, p_t)$, where the base point $p_t$ ($t \in [0,1]$) depends on $t$ continuously.

(Z3) The fundamental group of an irreducible curve of order $n$, possessing ordinary double points only, is cyclic of order $n$ ([130, Theorem 7]), see Remark 1.

Received May 23, 2006.
Revised May 14, 2007.
2000 *Mathematics Subject Classification.* 14H50, 20F36, 14B0, 14G32.
*Key words and phrases.* Zariski pair, fundamental group, Alexander invariants, Galois cover.
The two first authors are partially supported by MTM2007-67908-C02-01.