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A survey of complex Finsler geometry

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CONTENTS

§1. Introduction

Classically geometric problems on complex manifolds are investigated and solved by choosing appropriate Hermitian (or Kähler) metrics. In most cases the results depend, not just on the complex structure but on the choices of the Hermitian/Kähler metrics. On the other hand, naturally arisen *intrinsic* metrics are, almost always, not Hermitian but only Finsler in nature. The term "intrinsic" here refers to objects (or properties) that depend only on the complex structure of the manifold. The most well-known ones are the Kobayashi and the Caratheodory pseudometrics ([39], [41]) with the property that Kobayashi is the largest while Caratheodory is the smallest among all intrinsic pseudo-metrics. Essentially all intrinsic (pseudo) metrics arise as solutions of naturally posed extremal problems and, except in very special cases, only the Finsler character is preserved in the minimizing/maximizing process. This renders the deep and beautiful theory of Hermitian/Kähler geometry powerless in dealing with these metrics. Naturally, it is desirable to have a good differential geometric theory for Finsler metrics which, thanks to

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