Advanced Studies in Pure Mathematics 47-1, 2007
Asymptotic Analysis and Singularities
pp. 341–348

On the Stokes equation with Robin boundary condition

Yoshihiro Shibata and Rieko Shimada

§1. Introduction

Let $\Omega \subset \mathbb{R}^n$, $n \geq 2$ be a domain with boundary $\partial\Omega \in C^{2,1}$ and suppose one of the following case; Ω is a bounded domain, an exterior domain, a half-space or a perturbed half-space, *i.e.* there exists $R > 0$ such that $\Omega \setminus B_R = \mathbb{R}^n_+ \setminus B_R$, where $B_R = \{x \in \mathbb{R}^n \,||x| < R\}$, $\mathbb{R}^n_+ = \{x = (x_1, \cdots, x_n) \in \mathbb{R}^n \,|\, x_n > 0\}$. We consider the following Stokes system with Robin boundary condition.

(1.1)
$$\begin{cases} u_t - \operatorname{Div} T(u, \pi) = f, \ \operatorname{div} u = g = \operatorname{div} \tilde{g} & \text{in}\, \Omega \times (0, T_0) \\ \nu \cdot u|_{\partial\Omega} = 0, \ \alpha u + T(u)\nu - (T(u)\nu, \nu)\nu|_{\partial\Omega} = h \\ \quad u|_{t=0} = u_0 \end{cases}$$

where α is a non-negative constant, $u(x, t) = (u_1, \ldots, u_n)$ is a velocity field and $\pi(x, t)$ is a scalar pressure. Here and hereafter ν denotes the unit outer normal to $\partial\Omega$, $T(u, \pi)$ is the stress tensor defined by the formula:

$$T(u, \pi) = D(u) - \pi I \tag{1.2}$$

where $D(u)$ is the deformation tensor of the velocity with elements $D_{ij}(u) = \partial_i u_j + \partial_j u_i$. It is obvious that the boundary condition in (1.1) is equivalent to the following condition

$$\nu \cdot u|_{\partial\Omega} = 0, \ \alpha u + D(u)\nu - \big(D(u)\nu, \nu\big)\nu|_{\partial\Omega} = h. \tag{1.3}$$

So we may consider the condition (1.3) in stead of the boundary condition of (1.1). The cases when $\alpha = \infty$ and $\alpha = 0$ are corresponding to

Received October 31, 2005.
Revised January 11, 2006.