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## Characteristic classes of (pro)algebraic varieties

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## Dedicated to Jean-Paul Brasselet on the occasion of his sixtieth birthday

#### §1. Introduction

Various characteristic classes of singular varieties have been introduced and studied. One of them is the so-called Chern-Schwartz-MacPherson class. Its unique existence was conjectured by P. Deligne and A. Grothendieck and it was affirmatively solved by R. MacPherson. This characteristic class is a fundamental and important characteristic class from the viewpoint of investigation of other characteristic classes.

In this paper, in the first half we make a quick survey on three interesting characteristic classes of singular varieties with a naïve motivation of constructing a "singular version" of the so-called generalized Hirzebruch–Riemann–Roch theorem behind, and state a "unification" theorem concerning these three characteristic classes and its bivariant-theoretic version. And in the latter half we make a quick survey on characteristic classes of proalgebraic varieties, which are very much related to motivic measure and motivic integration.

# §2. Hirzebruch–Riemann–Roch and Grothendieck–Riemann–Roch

A characteristic class of a vector bundle over a topological space X is defined to be a map from the set of isomorphism classes of vector bundles over X to the cohomology group (ring)  $H^*(X;\Lambda)$  with a coefficient ring

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