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Motivic sheaves and intersection cohomology

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We propose a motivic refinement of a result in [BBFGK]. The formulation involves the notion of intersection Chow group, introduced by A. Corti and the author.

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§1. Intersection Chow groups and lifting theorems

We consider quasi-projective varieties over $k = \mathbb{C}$. For a quasi-projective variety Z, $\mathrm{CH}_s(Z)$ denotes the Chow group of s-cycles on Z tensored with $\mathbb{Q}$; if Z is smooth, $\mathrm{CH}^r(Z) := \mathrm{CH}_{\dim Z - r}(Z)$. We consider only constructible sheaves of $\mathbb{Q}$-vector spaces. The singular (co-)homology, Borel-Moore homology, and intersection cohomology are those with $\mathbb{Q}$-coefficients.

Relative canonical filtration.

The study of filtration on the Chow group of a smooth projective variety was started by Bloch and continued by several people; of most relevance to us now are the works of Beilinson, Murre and Shuji Saito. Beilinson explained the filtration in terms of the conjectural framework of mixed motives. Murre proposed a set of conjectures, Murre's conjectures, on a decomposition of the diagonal class in the Chow ring of self-correspondences; he relates the decomposition to the filtration of Chow groups.

For X a smooth projective variety, its Chow group of codimension r cycles $\mathrm{CH}^r(X)$ should have a filtration $F^\bullet$ such that $\mathrm{CH}^r(X) = F^0\,\mathrm{CH}^r(X)$, $F^1\,\mathrm{CH}^r(X)$ is the homologically trivial part, $F^2\,\mathrm{CH}^r(X)$ is perhaps the kernel of Abel-Jacobi map, and so on. The subquotient $Gr_F^\nu\,\mathrm{CH}^r(X)$ should in some way be determined by cohomology $H^{2r-\nu}(X, \mathbb{Q})$.

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