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Polarized K3 surfaces of genus thirteen

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A smooth complete algebraic surface S is *of type K3* if S is regular and the canonical class K_S is trivial. A *primitively polarized* K3 surface is a pair (S, h) of a K3 surface S and a primitive ample divisor class $h \in \operatorname{Pic} S$. The integer $g := \frac{1}{2}(h^2) + 1 \geq 2$ is called the *genus* of (S, h). The moduli space of primitively polarized K3 surfaces of genus g exists as a quasi-projective (irreducible) variety, which we denote by $\mathcal{F}_g$. As is well known a general polarized K3 surface of genus $2 \leq g \leq 5$ is a complete intersection of hypersurfaces in a weighted projective space: $(6) \subset \mathbf{P}(1112), (4) \subset \mathbf{P}^3, (2) \cap (3) \subset \mathbf{P}^4$ and $(2) \cap (2) \cap (2) \subset \mathbf{P}^5$.

In connection with the classification of Fano threefolds, we have studied the system of defining equations of the projective model $S_{2g-2} \subset \mathbf{P}^g$ and shown that a general polarized K3 surface of genus g is a complete intersection with respect to a homogeneous vector bundle $\mathcal{V}_{g-2}$ (of rank $g-2$) in a g-dimensional Grassmannian $G(n, r)$, $g = r(n-r)$, in a unique way for the following six values of g:

g	6	8	9	10	12	20
r	2	2	3	5	3	4
$\mathcal{V}_{g-2}$	$3\mathcal{O}_G(1) \oplus \mathcal{O}_G(2)$	$6\mathcal{O}_G(1)$	$\bigwedge^2 \mathcal{E} \oplus 4\mathcal{O}_G(1)$	$\bigwedge^4 \mathcal{E} \oplus 3\mathcal{O}_G(1)$	$3\bigwedge^2 \mathcal{E} \oplus \mathcal{O}_G(1)$	$3\bigwedge^2 \mathcal{E}$

Here $\mathcal{E}$ is the universal quotient bundle on $G(n, r)$. See [4] and [5] for the case $g = 6, 8, 9, 10$, [6, §5] for $g = 20$ and §3 for $g = 12$.

By this description, the moduli space $\mathcal{F}_g$ is birationally equivalent to the orbit space $H^0(G(n, r), \mathcal{V}_{g-2})/(PGL(n) \times Aut_{G(n,r)}\, \mathcal{V}_{g-2})$ and

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