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Vanishing theorem on the pointwise defect of a rational iteration sequence for moving targets

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§1. Introduction

Let f be a rational map, i.e., a holomorphic endomorphism of the Riemann sphere $\hat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$, of degree $d > 1$. The k times iteration of f is denoted by f^k for $k \in \mathbb{N}$.

The Nevanlinna theory for sequences was first studied in [19], [2], [8] and [10], and recently, motivated by complex dynamics, studied in [18], [16] and [15], where the sequence of rational maps correspond to a transcendental meromorphic function. Hence the following definition is natural:

Definition 1.1 (Picard exceptional value). The point $a \in \hat{\mathbb{C}}$ is called a *Picard exceptional value* of $\{f^k\}$ if

$$\# \bigcup_{k \in \mathbb{N}} f^{-k}(a) < \infty.$$

The point $a \in \hat{\mathbb{C}}$ is a Picard exceptional value if and only if it is periodic of period at most two and a and $f(a)$ are critical of order $d - 1$. In particular, there exist at most two such values (cf. [9]), which is an analogue of the Picard theorem for transcendental meromorphic functions.

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