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Representations of nonnegative solutions for parabolic equations

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§1. Introduction

This paper is an announcement of results on integral representations of nonnegative solutions to parabolic equations, and gives a representation theorem which is general and applicable to many concrete examples for establishing explicit integral representations.

We consider nonnegative solutions of a parabolic equation

$$(1.1) \qquad (\partial_t + L)u = 0 \quad \text{in} \quad D \times (0, T),$$

where T is a positive number, D is a non-compact domain of a Riemannian manifold M, $\partial_t = \partial/\partial t$, and L is a second order elliptic operator on D. We study the problem:

Determine all nonnegative solutions of the parabolic equation (1.1). This problem is closely related to the Widder type uniqueness theorem for a parabolic equation, which asserts that any nonnegative solution is determined uniquely by its initial value. (For Widder type uniqueness theorems, see [1], [5], [10], [13] and references therein.) We say that **[UP]**(i.e., uniqueness for the positive Cauchy problem) holds for (1.1) when any nonnegative solution of (1.1) with zero initial value is identically zero. When [UP] holds for (1.1) the answer to our problem is extremely simple: for any nonnegative solution of (1.1)there exists a

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