Advanced Studies in Pure Mathematics 43, 2006
Singularity Theory and Its Applications
pp. 73–84

tt^* geometry and mixed Hodge structures

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tt^* geometry is a generalization of variation of Hodge structures (section 2). Also the nilpotent orbits of Schmid and the relation to polarized mixed Hodge structures generalize; part of this is still a conjecture (section 3). tt^* geometry turns up in the unfoldings of holomorphic functions with isolated singularities (section 4).

This short paper is an introduction and a survey. It gives definitions, results, conjectures and references, but no proofs. It follows closely a talk which was given at the conference on *Singularity theory and its applications* (MSJ-IRI2003) in Sapporo, Japan, on 16-25 September 2003.

§1. Motivation and history

An isolated hypersurface singularity comes equipped with a polarized mixed Hodge structure (PMHS) on the middle cohomology of a Milnor fiber [St]. If one considers a semiuniversal unfolding with base space M of such a singularity, one obtains a variation of PMHS's on a subspace of M, the μ-constant stratum. But in fact, the variation of PMHS's extends to a variation of a more general structure on the whole base space.

This structure is called tt^* geometry. The purpose of this paper is to define it and discuss it first in an abstract setting and then in the case of singularities.

tt^* geometry was established more than 10 years ago in the work of Cecotti and Vafa [CV1][CV2][CV3]. They considered moduli spaces of $N = 2$ supersymmetric field theories. A distinguished class of these field theories, the Landau-Ginzburg models, is closely related to singularities. Especially, the unfoldings of quasihomogeneous singularities were studied by Cecotti and Vafa. Their work deserves much more attention from the singularity community.

Received March 31, 2004.
Revised August 18, 2004.