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Prolongation of holomorphic vector fields on a tube domain and its applications

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Introduction

In general, in the study of the holomorphic equivalence problem for complex manifolds, that is to say, the problem of investigating what happens when two complex manifolds are biholomorphically equivalent, it is one of standard ways to direct our attention to biholomorphic invariant objects. As a typical and good example of such objects, we have holomorphic automorphism groups. In fact, when Poincaré showed that a ball and a polydisk in $\mathbf{C}^2$ are not biholomorphically equivalent, he looked at their holomorphic automorphism groups, and showed that the dimensions do not coincide. One of the foundations of observations like this is the pioneer result of H. Cartan that the holomorphic automorphism group of a complex bounded domain has the structure of a Lie group.

Now, when a holomorphic automorphism group has the structure of a Lie group, what advantage do we have? It seems that one advantage is that conjugacy theorems in Lie group theory can be applied. The conjugacy theorems are very powerful tools, and if they can be applied well, splendid achievements are produced. But, in order to apply the conjugacy theorems, we need to know a lot about the Lie group structure of a holomorphic automorphism group. So, since Lie algebra provides much useful information about Lie group, we are led to turning our eyes to the Lie algebra of complete holomorphic vector fields corresponding to the Lie algebra of a holomorphic automorphism group. Then, in the process of investigating such Lie algebras, we often come up against the problem of completeness, or the fundamental problem of judging whether a vector field is complete or not. In general, a judgement on the completeness of a vector field is very difficult to deal with. Actually, given a vector field, the problem of whether its integral curve is lengthened to

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