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Generalization of a precise L^2 division theorem

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§ Introduction

The purpose of this article is to generalize the following.

Theorem 1 (cf. [O-3]). *Let D be a bounded pseudoconvex domain in $\mathbf{C}^n$ and let $z = (z_1, \dots, z_n)$ be the coordinate of $\mathbf{C}^n$. Then there exists a constant C depending only on the diameter of D such that, for any plurisubharmonic function φ on D and for any holomorphic function f on D satisfying*

$$\int_D |f(z)|^2 e^{-\varphi(z)} |z|^{-2n} \, d\lambda < \infty \tag{1}$$

there exists a vector valued holomorphic function $g = (g_1, \dots, g_n)$ on D satisfying

$$f(z) = \sum_{i=1}^{n} z_i g_i(z) \tag{2}$$

with

$$\int_D |g(z)|^2 e^{-\varphi(z)} |z|^{-2n+2} \, d\lambda \leq C \int_D |f(z)|^2 e^{-\varphi(z)} |z|^{-2n} \, d\lambda. \tag{3}$$

Here $d\lambda$ denotes the Lebesgue measure.

We generalize this in order to establish an understanding that the measure $e^{-\varphi}|z|^{-2n} \, d\lambda$ in (1) consists of three parts, i.e. $e^{-\varphi(z)}$ for any plurisubharmonic function φ, $|z|^{-2}$ as the quotient fiber metric associated to the morphism $g \mapsto \sum z_i g_i$, and $|z|^{-2n+2} \, d\lambda$ as the residue of a volume form on $(D \setminus \{0\}) \times \mathbf{P}^{n-1}$ with respect to the embedding of $D \setminus \{0\}$ by $z \mapsto (z, [z])$, where $[z] = (z_1 : \cdots : z_n)$.

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