Advanced Studies in Pure Mathematics 42, 2004
Complex Analysis in Several Variables
pp. 159–164

The Bergman kernel of Hartogs domains and transformation laws for Sobolev-Bergman kernels

Gen Komatsu

Introduction

If we consider the Bergman kernel of strictly pseudoconvex domains, we can discuss a scalar invariant theory associated with CR geometry of the boundaries. This is Fefferman's program proposed in [3] and then developed in [6], [10], [1], [11], [8] and others. What will happen if the Bergman kernel is replaced by reproducing kernels associated with spaces of holomorphic functions contained in L^2 Sobolev spaces? Let us restrict ourselves to the case where the Sobolev order is a half integer $s/2$ ($s \in \mathbb{Z}$). The case $s = 0$ corresponds to the Bergman kernel. The case $s = 1$ corresponds to the Szegö kernel, and the invariant theory is essentially the same as that of the Bergman kernel ([10], [11]). The situation changes with the signature of this s. More precisely, it is at first necessary that the inner product of the Hilbert space which admits the reproducing kernel must satisfy a transformation law under biholomorphic mappings. Existence of such an inner product is obvious when $s \leq 0$ ($s \in \mathbb{R}$), whereas it is unknown for $s > 0$ except for $s = 1$. Next, boundary invariants will be contained in the singularity of the reproducing kernel, and if the singularity is of the same type as that of the Bergman kernel ([3], [2]) then in particular $s \geq 0$ is necessary ([9]). This fact suggests that the type of the singularities of the reproducing kernels for $s < 0$ are different from that of the Bergman kernel. Is it possible to avoid considering such new singularities? In what follows, we shall give an almost affirmative answer by considering Hartogs domains and Hirachi's formulation in [8] of a biholomorphic transformation law for local defining functions of strictly pseudoconvex domains.

Received March 14, 2002

Research partially supported by Grant-in-Aid for Scientific Research, Ministry of Education, Science and Culture.