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Ideals of multipliers

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Ideals of multipliers were introduced in [8] to find conditions on domains in complex manifolds under which subellipticity of the $\bar{\partial}$-Neumann problem holds. Similar ideals were used to study subellipticity on of $\Box_b$ on CR manifolds (see [9]). In [10] such ideals are used to study the situation when subellipticity breaks down but regularity still holds. Ideals of holomorphic multipliers in a somewhat different context have been used by Nadel (see [15]) and by Siu (see [16]) to prove global theorems in algebraic geometry. Here we will be concerned with the ideals that arise in the study of local regularity. We will briefly explain the use of subelliptic estimates then we define local and microlocal multipliers and show how to use them to derive subelliptic estimates. We also discuss the use of subelliptic multipliers when subellipticity fails. Finally we show how subelliptic multipliers give rise to invariants of complex analytic varieties.

§1. Definitions

A **CR manifold** is a compact C^∞ manifold M of dimension $2n+1$ endowed with an **integrable CR structure** which consists of a subbundle $T^{1,0}(M)$ of the complexified tangent bundle $\mathbb{C}T(M)$ satisfying the following. The complex fiber dimension of $T^{1,0}(M)$ is n,

$$T^{1,0}(M) \cap \overline{T^{1,0}(M)} = \{0\},$$

and if L and L' are local sections of $T^{1,0}(M)$ then $[L, L'] = LL' - L'L$ is also a local section of $T^{1,0}(M)$.
Let $\mathcal{A}_b^{p,q}$ denote the (p,q)-forms on M, let

$$\bar{\partial}_b : \mathcal{A}_b^{p,q} \to \mathcal{A}_b^{p,q+1}$$

denote the corresponding exterior derivative, and let $\bar{\partial}_b^* : \mathcal{A}_b^{p,q} \to \mathcal{A}_b^{p,q-1}$ denote the L_2 adjoint of $\bar{\partial}_b$.

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